Venue ____________________

Student Number |__|__|__|__|__|__|__|__|

Research School of Economics

EXAMINATION

Semester 2, 2023 — Deferred exam

ECON2125/ECON6012_Semester 2 Optimisation for Economics and Financial Economics

This paper is for ANU and ANU students.

Examination Duration:	180 minutes
Reading Time:	15 minutes
Exam Conditions:	
(No electronic aids are permitted e.g. laptops, phones)	
Materials Permitted In The Exam Venue:	Calculator (non programmable)
Materials To Be Supplied To Students:	1 x 20 page
Instructions To Students:	See next page

INSTRUCTIONS TO STUDENTS

- Read the questions carefully.
- Questions are worth different amount of marks given in parenthesis. Sub-questions in each questions are of equal value.
- To maximize your marks, explain all the steps in your arguments.
- If any part of the question seems missing or ambiguous, state clearly the way you interpret it, and carry of with your answer.
- In solving the questions, you can use any fact from the lecture materials without proof, unless specifically asked to give details. In either case, you should clearly state the relevant fact.
- You do not need to do the questions in order, as long as you clearly mark in your answer sheets which question you are addressing.

QUESTIONS

Question 1. (*10 marks*)

Consider function $f\colon [a,b] \to \mathbb{R}$ defined by $f(x) = -\frac{1}{x}e^x$. Find the minimizer(s) and the maximizer(s) of this function on $D = [0,3]$. Explain your reasoning.

Question 2. (*10 marks*)

Find trace and determinant of the following matrix for each $\lambda \in \{-1, 0, 1\}$. Which values of λ make the matrix singular (non-invertible)?

$$\begin{pmatrix} 1, & 0, & 1 \\ 1, & 2, & 3 \\ 0, & 1, & \lambda \end{pmatrix}$$

Question 3. (*10 marks*)

Find an example of a nonlinear function $f\colon D \subset \mathbb{R} \to \mathbb{R}$ that has has exactly two minimizers and no maximizer. Remember to define both the function and its domain. Explain why the function has this property.

Question 4. (*10 marks*)

Let $g\colon \mathbb{R}^N \to \mathbb{R}$ be defined by $g(x) = -\|x\|$.

- Is g a bijection? Explain.
- Is g a concave? If yes, strictly or weakly? Explain.

Question 5. (*10 marks*)

Consider a correspondence $\gamma\colon \mathbb{R} \to 2^{\mathbb{R}}$ given by

$$\gamma(x) = \left\{ y \in \mathbb{R}\colon |x| \leq \sqrt{|y|} \right\}$$

Make a sketch of the graph of correspondence γ. Is γ continuous? Explain.

Question 6. (*10 marks*)

For a function $f\colon \mathbb{R}^2 \ni (x,y) \mapsto 2x^2 + y^4 - 2xy \in \mathbb{R}$ find the stationary points and determine whether they are local maxima or minima.

Question 7. (*20 marks*)

An infinite horizon deterministic wealth draw-down problem is given by the following Bellman equation

$$V(m) = \max_{c \in [0,m]} \big\{ u(c) + \beta V\big((m-c)(1+r)\big) \big\},$$

where $m \in [0, M] \subset \mathbb{R}$ denotes the current level of wealth, c is consumption, $r \in (0,1)$ is the interest rate, and $\beta \in (0,1)$ is the discount factor. Assume that M is a finite upper bound. Let $u(\cdot)$ denote a strictly concave continuous utility function.

Assuming that the optimal consumption choice function $c^\star(m)$ is differentiable, show that away from the corner solutions it is characterized by

$$u'\big(c^\star(m)\big) = \beta(1+r)u'\big(c^\star\big([m - c^\star(m)](1+r)\big)\big).$$

Find the values of the parameters of the problem which yield optimal consumption to be constant over time, i.e. perfect consumption smoothing.

Question 8. (*20 marks*)

Find local minimizers and maximizers of the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = x^2 + y^2$ on the set

$$D = \{(x,y)\colon (1-x)^3 \geq y^2 \text{ and } |y| - 1 \leq 1\}.$$

Use first and second order conditions to support your answer.

END OF EXAMINATION