Comments on Exam Questions

Mainly small proofs/arguments requiring only a few steps of logic

In general, good answers will

- include relevant definitions

- use relevant facts from slides

- avoid long and difficult calculations — there's probably an easier way

Use of external theorems is discouraged

- You won't need them

- Don't tell me it's true because you saw it in a book

Sample question, worth five marks:

Q: Let $\mathbf{A}$ be any matrix. Show that the symmetric matrix $\mathbf{A}'\mathbf{A}$ is nonnegative definite

What is a good answer to this question?

A1 I love Kung Fu

- Mark: 0/5
- Why: Irrelevant

A2 $N \times N$ symmetric matrix $\mathbf{B}$ is nonnegative definite if $\mathbf{x'Bx} \geq 0$ for any $N \times 1$ vector $\mathbf{x}$. I don't know the rest.

- Mark: 2/5
- Why: Gave the relevant definition

A3 $N \times N$ symmetric matrix $\mathbf{B}$ is nonnegative definite if $\mathbf{x'Bx} \geq 0$ for any $N \times 1$ vector $\mathbf{x}$. Strictly concave functions have unique minima. A set is a collection of objects. Sharks continue to swim while sleeping.

- Mark: 1/5
- Why: One relevant definition cancelled out by other noise

A4. By definition, an $N \times N$ symmetric matrix $\mathbf{B}$ is nonnegative definite if

$$\mathbf{x}'\mathbf{B}\mathbf{x} \geq 0 \text{ for any } N \times 1 \text{ vector } \mathbf{x} \qquad (\star)$$

Let $\mathbf{B} := \mathbf{A}'\mathbf{A}$ and fix any such $\mathbf{x}$. By the rules of transposes we have

$$\mathbf{x}'\mathbf{B}\mathbf{x} = \mathbf{x}'\mathbf{A}'\mathbf{A}\mathbf{x} = (\mathbf{A}\mathbf{x})'(\mathbf{A}\mathbf{x}) \geq 0$$

Here last equality holds because, for any vector $\mathbf{y}$,

$$\mathbf{y}'\mathbf{y} = \sum_{n=1}^{N} y_n^2 \geq 0$$

This confirms $(\star)$

- Mark: 5/5
- Why: Correct and crystal clear